Activity prepared by the KCL EPAP group
Experimental Particle and Astroparticle Physics

The Sun is a massive neutrino (ν) generating nuclear reactor

How many of those neutrinos reach you?

In this activity you will be working with very large numbers and very small probabilities to calculate how many of the neutrinos created in the Sun might pass through your own body.

Solutions

These are example answers but you will probably have made different estimations and will have slightly different numerical answers.

Check to see if you got the right order of magnitude.

<u>Mathematical concepts used:</u>

- Place value and indices
- Area of a sphere
- Speed = distance / time
- Probability
- Estimation

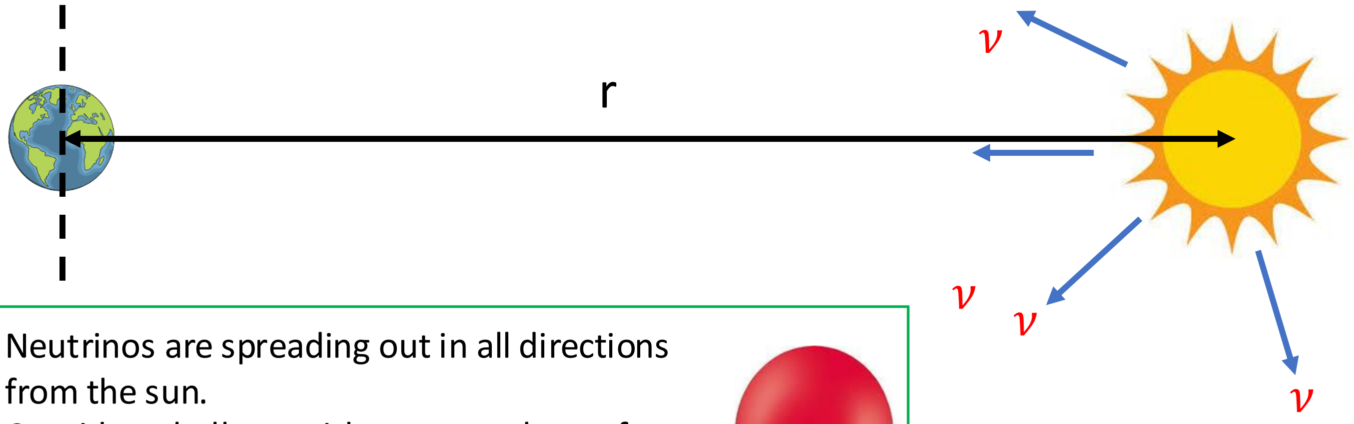
1. Calculate the neutrino flux at Earth

- The Sun produces 10^{35} neutrinos per second!
  - These are emitted in all directions (isotropically) so consider them spreading out on the surface of a sphere
- The Sun is 152 million kilometers away from Earth

Flux, ϕ = rate of particles passing through a given area ($1 m^2$) in a given time (1s) .

Surface area of a sphere
$A = 4\pi r^2$

How many neutrinos per second pass through a $1m^2$ area here?

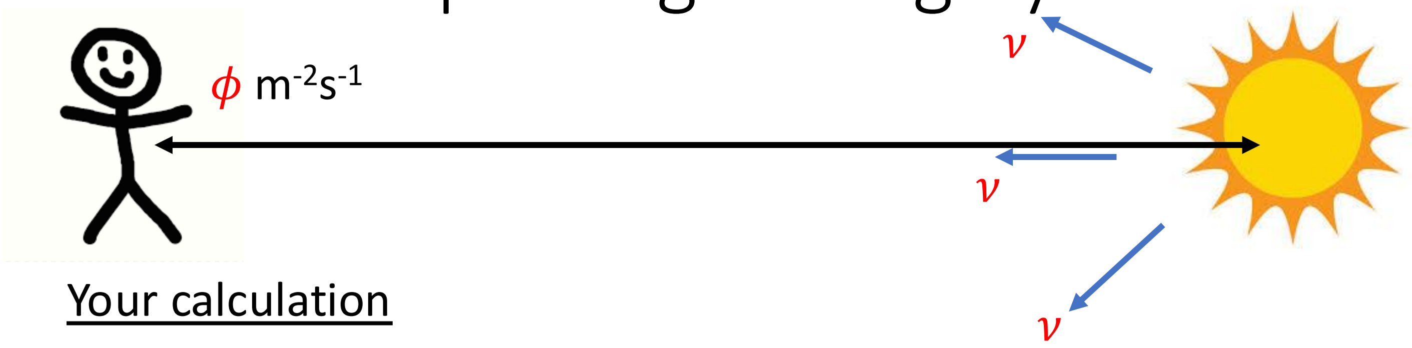


Neutrinos are spreading out in all directions from the sun.
Consider a balloon with spots on the surface – as you blow the balloon up there are the same number of spots but they get further away from the centre and more spread out.

Your calculation

Quantity	Symbol	Your Answer
Distance neutrinos travel from the Sun to the Earth	r	$= 1.52 \times 10^{11} m$
Area of sphere with radius r	A	$= 4\pi r^2$ $= 4\pi \times (1.52 \times 10^{11})^2$ $= 2.9 \times 10^{23} m^2$
$1m^2$ unit area on the surface of Earth as a fraction of area A	$f = \frac{1}{A}$	$= \frac{1}{2.9 \times 10^{23}}$ $= 3.44 \times 10^{-24} m^{-2}$
The neutrino flux at Earth is this fraction f times the total number of neutrinos produced in the sun each second	ϕ	$= 10^{35} \times 3.44 \times 10^{-24}$ $= 3.44 \times 10^{11} m^{-2} s^{-1}$

2. Calculate the number of neutrinos passing through you



<u>Your calculation</u>

Quantity	Symbol	Your Answer
Estimate your area	a	For simplicity, $a = 1m^2$
Number of neutrinos passing through you in one second	$N_1 = a \times \phi$	$= 1 \times 3.44 \times 10^{11}$ $= 3.44 \times 10^{11} s^{-1}$

Does it make a difference if you are standing up or lying down?

What matters is your area perpendicular to the direction of the Sun so if the Sun is overhead, the area the neutrinos can pass through will be bigger is you are lying down than standing up.

3. What is the chance that one of those neutrinos will interact in your body during your lifetime?

We can only 'see' neutrinos if they interact. The chance, of any single neutrino interacting with you is 1 in 10^{22}. $C = 10^{-22}$

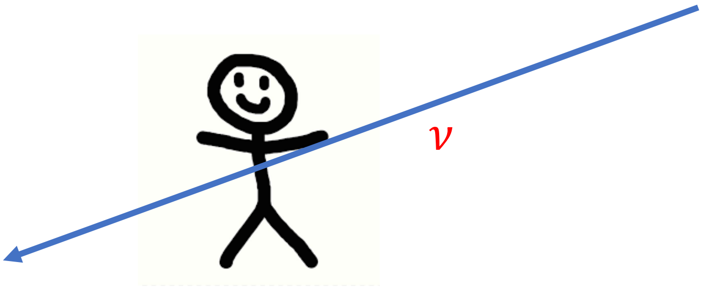
<u>Your calculation</u>

Quantity	Symbol	Your Answer
Estimate your lifetime in years	T_y	Let's estimate 80 years
Calculate your lifetime in seconds	T_s	$= 80 \times 365 \times 24 \times 60 \times 60$ $= 2.5 \times 10^9 s$
Calculate the number of neutrinos that will pass through you in that time	$N_{lifetime} = T_s \times N_1$	$= 2.5 \times 10^9 \times 3.44 \times 10^{11}$ $= 8.6 \times 10^{20}$
Calculate the probability that any of those neutrinos will interact	$P = N_{lifetime} \times C$	$= 2.5 \times 10^9 \times 3.44 \times 10^{11} \times 10^{-22}$ $= 0.086$ $= 8.6\%$

<u>Thinking Further:</u> How likely is it that somebody in your class has already had an interaction with a neutrino?

If there are 30 students aged 15 in a class, we get a total age of 450, *or* 5.6×80 *years*
We can multiply the probability to get $5.6 \times 8.6 = 48\%$ *chance that somebody in the class has already had an interaction with a solar neutrino.*

4. Calculate the probability that there is a neutrino inside you at any one instant



Neutrinos travel at virtually the speed of light, $c = 3 \times 10^8$ms^{-1}

Speed = distance / time

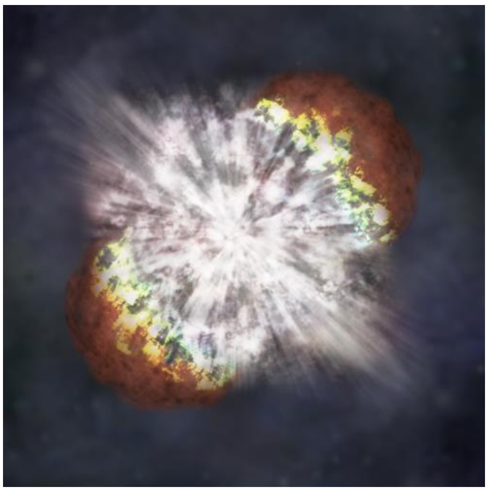
$$c = \frac{d}{t}$$

Quantity	Symbol	Your Answer
Estimate the average distance through your body	d	A path head-to-toe is longer than left-to-right. In these solutions we estimate that $d = 50cm = 0.5$m
Calculate the time each neutrino spends in you	$t = \frac{d}{c}$	$= \frac{0.5}{3 \times 10^8}$ $= 1.7 \times 10^{-9}s$ (remember this is an estimate so we don't need to be too accurate and your answer may differ but should be of the same magnitude)
Total time of neutrinos inside your body each second	$T_{total} = t \times N_1$	$= 1.7 \times 10^{-9} \times 3.44 \times 10^{11}$ $= 573$
This time, divided by 1s, gives the number of neutrinos inside you at any one time	$N_{instant} = \frac{T_{total}}{1}$	So at any one time there are over 500 neutrinos in your body so the probability that there will be at least 1 is 100%.

Values < 1 can be taken as the probability of a neutrino being inside you right now, Values > 1 are the expected number at any one time.

5. In the event of a supernova, how many neutrinos could pass through you?

A supernova is a powerful explosion when a star runs out of fuel. It could produce 10^{58} neutrinos in just a few seconds!



IK Pegasi B is the nearest know supernova candidate, located 150 light years away.

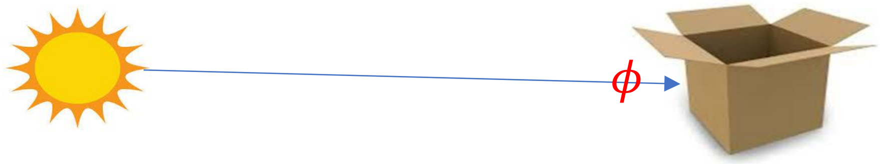
Candidate means a star that we think might go supernova

A light year is the distance that light travels in 1 year, $d = ct$
Remember $c = 3 \times 10^{8}$ ms^{-1}

Your Calculation

Quantity	Symbol	Your Answer
The distance between IK Pegasi and Earth in metres	d	$= ct$ $= 3 \times 10^{8} \times 365 \times 24 \times 3600$ $= 9.5 \times 10^{15} m$
Area of a sphere with this radius (ie at Earth's surface)	A	$= 4\pi r^2$ $= 4\pi \times (9.5 \times 10^{15})^2$ $= 1.1 \times 10^{33} m^2$
The number of neutrinos passing through 1m^2 of that sphere	ϕ	$= \frac{10^{58}}{1.1 \times 10^{33}}$ $= 8.9 \times 10^{24}$
Using your estimated area, a, calculate the number of neutrinos passing through you from the supernova	N_{SN}	$= a\phi$ $= 1 \times 8.9 \times 10^{24}$ So the probability of interacting with a neutrino if IK Pegasus B goes supernova is $8.9 \times 10^{24} \times 10^{-22} = 890$ ie hundreds of neutrinos would interact with you in a few seconds!

Thinking further: What volume box do you need to be sure there is always a neutrino from the Sun in it?



Your Calculation

This is tricky and requires some rearranging of equations.

The number of neutrinos passing through is $N = \phi \times A \times t$

The volume of a box is the area times the depth, $V = A \times d$

And we can use the velocity equation to get the depth in terms of time and velocity of the neutrino, $d = c \times t$

Substituting $V = A \times c \times t$ so $A \times t = \frac{V}{c}$

And substituting in the first equation gives $N = \phi \times \frac{V}{c}$

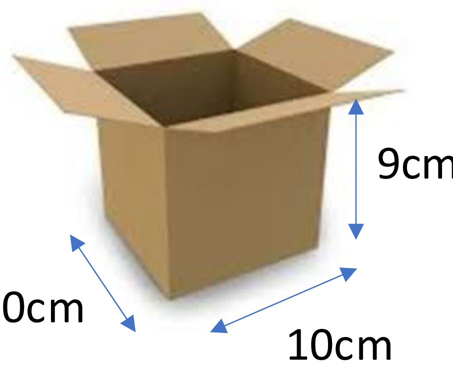
Rearranging that and setting $\mathrm{N} = 1$ gives

$$V = \frac{c}{\phi} = \frac{3 \times 10^8}{3.44 \times 10^{11}}$$

(using our solar neutrino flux value calculated in question 1)

$$= 8.7 \times 10^{-4} m^3 = 870 cm^3$$

So we could try a box that is around $10cm \times 10cm \times 9cm$!



Well done for completing this sheet! Please let us know how you found the exercise by filling in this form: https://forms.office.com/e/Rgsu2GbaRs